VI Semester B.A./B.Sc. Examination, May 2016
(Semester Scheme)
(2013-14 and Onwards) (NS) (F + R)
MATHEMATICS (Paper – VII)

Time : 3 Hours Max. Marks : 100

*Instruction : Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Find the locus of the point z, satisfying $|z-4| \geq 3$.

2) Evaluate $\lim_{z \to i}\left(\frac{z^2+1}{z^6+1}\right)$.

3) Show that $f(z) = \sin z$ is analytic.

4) Prove that $u = y^3 - 3x^2y$ is a harmonic function.

5) Find the invariant (fixed) points of the bilinear transformation $W = \frac{3z-5}{z+1}$.

6) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the line $y = x$.

7) State Liouville's theorem.

8) Evaluate $\int_C \frac{1}{z(z-2)}dz$, where C is the circle $|z| = 3$.

9) Evaluate $\int_C \left[\left(2y+x^2\right)dx + (3x-y)dy\right]$ along the curve $x = 2t$, $y = t^2 + 3$, where $0 \leq t \leq 1$.

10) Evaluate $\int_0^1 \int_0^1 \left(x^2+y^2\right)dydx$.

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11) Evaluate $\int_{0}^{2\pi}\int_{1}^{2} r^3 \cos^2\theta \sin^2\theta \, dr d\theta$.

12) Evaluate $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)} \, dxdy$ by changing into polar co-ordinates.

13) Evaluate $\int_{0}^{1}\int_{0}^{2}\int_{1}^{2} xyz^2 \, dxdydz$.

14) State Green's theorem in the plane.

15) Prove that $\text{div}\left(\text{curl } \vec{F}\right) = 0$, using Stoke's theorem.

16) Evaluate $\iint_S \left[(x+z)\hat{i} + (y+z)\hat{j} + (x+y)\hat{k}\right].\hat{n}\, ds$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ by using Gauss divergence theorem.

17) Define an interior point.

18) State Bolzano-Weistrass theorem.

19) Define a topological space.

20) Let $X = \{a, b\}$ and $\tau = \{X, \phi, \{a\}, \{b\}\}$ be a topology on X. Find τ neighbourhood of 'a'.

II. Answer **any four** questions : **(4×5=20)**

1) Show that locus of a point z, satisfying $\text{amp}\left(\frac{z-1}{z+2}\right) = \pi/3$ is a circle. Find its centre and radius.

2) Prove the Cauchy-Riemann equations in the polar form $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$.

3) Show that $f(z) = e^z$ is analytic and hence show that $f'(z) = e^z$.

4) Find the analytic function whose imaginary part is $e^{-y}(x \sin x + y \cos x)$.

5) Discuss the transformation $w = \sin z$.

6) Find the bilinear transformation which maps $z = 0, -i, -1$ onto $w = i, 1, 0$.

III. Answer **any two** questions. **(2×5=10)**

1) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where $c : |z| = 3$.

2) State and prove Cauchy's integral formula.

3) If a in any positive real number and c is the circle $|z| = 3$, show that

$$\int_C \frac{e^{2z}}{(z^2+1)^2} dz = \pi i(\sin a - a \cos a).$$

IV. Answer **any four** questions. **(4×5=20)**

1) Evaluate $\int_C [3x^2 dx + (2xz - y)dy + zdz]$ along the line joining (0, 0, 0) and (2, 1, 3).

2) Evaluate $\iint_R y\,dxdy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

3) Evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} xy\,dxdy$ by changing the order of integration.

4) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.

5) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \dfrac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$.

6) Evaluate $\iiint_R xyzdxdydz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$ by changing it to spherical polar co-ordinates.

V. Answer **any two** questions. **(2×5=10)**

1) State and prove Green's theorem in the plane.

2) Evaluate $\iint_S \left(x\hat{i}+y\hat{j}+z^2\hat{k}\right).\hat{n}ds$, where S is the closed surface bounded by the cone $x^2+y^2=z^2$ and the plane $z=1$, using divergence theorem.

3) Evaluate by Stoke's theorem $\oint_C (\sin zdx-\cos xdy+\sin ydz)$. Where C is the boundary of the rectangle $0 \le x \le \pi,\ 0 \le y \le 1, z=3$.

VI. Answer **any two** questions. **(2×5=10)**

1) Prove that the union of any number of open subsets of R^2 is open.

2) Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {a,b}} then show that τ is a topology on X.

3) Let A and B be any two subsets of the topological space X, then prove that

i) If $A \subset B \Rightarrow \overline{A} \subset \overline{B}$

ii) $\left(\overline{A \cup B}\right) = \overline{A} \cup \overline{B}$.

4) Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} be a topology for X. If β = {{a}, {b}, {c}} then show that β is a base of τ.