NS – 309

V Semester B.A./B.Sc. Examination, Nov./Dec. 2016
(Semester Scheme)
(Fresh) (CBCS) (2016-17 Onwards)
MATHEMATICS – V

Time : 3 Hours | Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) In a ring $(R, +, \cdot)$, prove that $(-a)\cdot(-b) = a\cdot b;\ \forall\ a, b \in R$.

   b) Define subring of a ring. Give an example.

   c) Give an example of

      i) Commutative ring without unity

      ii) Non-commutative ring with unity.

   d) Find the unit vector normal to the surface $xy^3z^2 = 4$ at $(-1, -1, 2)$.

   e) Find the divergence of $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

   f) Prove that $E = e^{hD}$.

   g) Write Lagranges Interpolation Formula.

   h) Evaluate $\int_0^1 \frac{dx}{1+x}$ by Simpson's $\frac{3}{8}^{th}$ rule.

   where

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
y = f (x)	1	0.8571	0.75	0.6667	0.6	0.5455	0.5

P.T.O.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) Show that the necessary and sufficient conditions for a non-empty subset S of a ring R to be a subring of R are

   i) $a \in S, b \in S \Rightarrow a - b \in S$

   ii) $a \in S, b \in S \Rightarrow ab \in S$.

   b) Prove that every field is an Integral Domain.

OR

3. a) Show that the set of all matrices of the form $\left\{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \middle/ a, b \in R\right\}$ is a non-commutative ring without unity with respect to addition and multiplication of matrices.

   b) Fill all the principal ideals of a ring $R = \{0, 1, 2, 3, 4, 5\}$ w.r.t. $+_6$ and $\times_6$.

4. a) Prove that $(Z_7, +_7, \times_7)$ is a commutative ring with unity. Is it a Integral Domain ?

   b) State and prove fundamental theorem of homomorphism.

OR

5. a) Prove that a commutative ring with unity is a field if it has no proper ideals.

   b) Prove that the mapping $f : (Z, +, \times) \to (2Z, +, *)$ where $a * b = \frac{ab}{2}$ defined by $f(x) = 2x, \forall x \in Z$ is an isomorphism.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Find the directional derivative of $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point $(1, 1, -1)$ in the direction of $2\hat{i} - \hat{j} + \hat{k}$.

   b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, prove that

   i) $\nabla r^n = n\, r^{n-2}\, \vec{r}$ ii) $\nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$.

OR

7. a) Show that the surfaces $4x^2y + z^3 = 4$ and $5x^2y - 2yz = 9x$ intersect orthogonally at the point (1, – 1, 2).

b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\nabla^2\left(\mathrm{div}\left(\frac{\vec{r}}{r^2}\right)\right) = \frac{2}{r^4}$.

8. a) If $\vec{F}$ = grad $(x^3 + y^3 + z^3 - 3xyz)$, find div$\vec{F}$ and curl $\vec{F}$.

b) If ϕ is scalar point function and $\vec{F}$ is vector point function then curl $(\phi\vec{F}) = \phi$ curl $\vec{F}$ + (grad ϕ) $\times \vec{F}$.

OR

9. a) If $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$, find a, b, c such that $\vec{F}$ is irrotational then find ϕ such that $\vec{F} = \nabla\phi$.

b) Prove that curl (curl $\vec{f}$) = grad (div $\vec{f}$) – $\nabla^2 \vec{f}$.

PART – D

Answer **two full** questions. **(2×10=20)**

10. a) Find a cubic polynomial which takes the following data :

x	0	1	2	3
f (x)	1	2	1	10

b) Find f(1.4) from the following data.

x	1	2	3	4	5
f (x)	1	8	27	64	125

using difference table.

OR

11. a) Evaluate $\Delta(e^{3x}\log 4x)$.

b) Find f(7.5) from the following data.

x	7	8	9	10
f (x)	3	1	1	9

using difference table.

12. a) Using Newton's divided difference formula find f(3) from the given data.

x	0	1	2	5
f (x)	2	3	12	147

b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}^{th}$ rule.

OR

13. a) Using Lagranges interpolation formula find f(2) from the following data.

x	0	1	3	4
f (x)	5	6	50	105

b) Using Simpson's $\frac{1}{3}^{rd}$ rule, evaluate $\int_0^{0.6} e^{-x^2} dx$.

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(Fresh) (CBCS) (2016 – 17 and Onwards)
MATHEMATICS – VI

Time : 3 Hours   Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions.   **(5×2=10)**

1. a) Write Euler's equation when f is independent of y.

   b) Show that the functional $I = \int_{x_1}^{x_2} (y^2 + x^2 y^1)\, dx$ assumes extreme values on the straight line y = x.

   c) Define geodesic on a surface.

   d) Evaluate $\int_C (5x\,dx + y\,dy)$ where C is the curve, $y = 2x^2$ from (0, 0) to (1, 2).

   e) Evaluate $\int_0^{\pi} \int_0^{a \ln y} y\,dx\,dy$.

   f) Evaluate $\int_0^1 \int_0^x \int_0^z dy\,dz\,dx$.

   g) Show that the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab using Green's theorem.

   h) Evaluate using Stoke's theorem $\oint_C (yz\,dx + zx\,dy + xy\,dz)$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

P.T.O.

PART – B

Answer **two full** questions : **(2×10=20)**

2. a) Prove that the necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y')\, dx$ with $y(x_1) = y_1$ and $y(x_2) = y_2$ to be an extremum is $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

   b) Find the geodesic on a plane.

OR

3. a) Show that the extremal of $I = \int_{x_1}^{x_2} \sqrt{y\,(1 + (y')^2}\, dx$ is a parabola.

   b) Find the extremal of the functional $I = \int_0^1 \sqrt{1 + (y')^2}\, dx$ with $y(0) = 1$ and $y(1) = 2$.

4. a) Find the shape of a chain which hangs under gravity between two fixed points.

   b) Find the extremal of the functional $\int_0^1 \left[(y')^2 + x^2\right] dx$ subject to constraint $\int_0^1 y\, dx = 2$ and having end conditions $y(0) = 0$, $y(1) = 1$.

OR

5. a) Find the function y which makes the integral $I = \int_{x_1}^{x_2} [y^2 + 4(y')^2]\, dx$ an extremum.

   b) Find the extremal of the functional $I = \int_0^{\pi} \left[(y')^2 - y^2\right] dx$ with $y(0) = 0$ and $y(\pi) = 1$ and subject to the constraint $\int_0^{\pi} y\, dx = 1$.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Evaluate $\int_C (x+y+z)\,ds$ where C is the line joining the points (0, 1, 0) and (1, 2, 3).

   b) Evaluate $\iint_A (4x^2 - y^2)\,dxdy$, where A is the area bounded by the lines $y = 0$, $y = x$ and $x = 1$.

OR

7. a) Evaluate $\int_0^\infty \int_0^\infty x\,e^{-x^2/y}\,dx\,dy$, by changing the order of integration.

   b) Find the area bounded by the arc of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in first quadrant.

8. a) Evaluate $\int_0^1 \int_0^{x^2} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$.

   b) Evaluate $\iint_R \frac{x^2y^2}{x^2+y^2}\,dxdy$ using polar co-ordinates, where R is the annular region between the circles $x^2 + y^2 = 2$ and $x^2 + y^2 = 1$.

OR

9. a) Find the volume bounded by the surface $z = a^2 - x^2$ and the planes $x = 0$, $y = 0$, $z = 0$ and $y = b$.

   b) If R is the region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$, show that $\iiint_R z\,dxdydz = \frac{1}{24}$.

PART – D

Answer **two full** questions : **(2×10=20)**

10. a) State and prove Gauss' Divergence Theorem.

b) Evaluate using Green's theorem for $\oint_C [xy\,dx + yx^2 dy]$, where C is the curve enclosing the region bounded by the curve $y = x^2$ and the line $y = x$.

OR

11. a) Verify Green's theorem in the plane for $\oint_C \left[(x^2 - xy^3)\,dx + (y^2 - 2xy)\,dy\right]$, where C is the square with vertices (0, 0), (2, 0), (2, 2) and (0, 2).

b) Evaluate $\iint_S \vec{F}\cdot\hat{n}ds$ using divergence theorem where $\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$ and S is the closed surface bounded by planes z = 0, z = 1 and the cylinder $x^2 + y^2 = 4$.

12. a) Verify Stokes theorem for $\vec{F} = 2y\hat{i} + 3x\hat{j} - z^2\hat{k}$; C is the boundary of the upper half of the surface of the sphere $x^2 + y^2 + z^2 = 9$.

b) Evaluate using Gauss' divergence theorem $\iint_S \vec{F}\cdot\hat{n}ds$, where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the total surface of the rectangular parallelopiped bounded by the planes x = 0, y = 0, z = 0, x = 1, y = 2, z = 3.

OR

13. a) Evaluate $\oint_C \vec{F}\cdot d\vec{r}$, using Stoke's theorem where $\vec{F} = (y - z + 2)\,\hat{i} + (yz)\,\hat{j} - (xz)\,\hat{k}$ taken over the surface S of the cube $0 \le x \le 2,\ 0 \le y \le 2,\ 0 \le z \le 2$.

b) By using Green's theorem evaluate $\oint_C [(3x - y)\,dx + (2x + y)\,dy]$ where C is the circle $x^2 + y^2 = a^2$.
