



SN – 349

I Semester B.A./B.Sc. Examination, November/December 2017
(CBCS) (F+R) (2014 –15 & Onwards)
MATHEMATICS – I



Time : 3 Hours

Max. Marks : 70

Instruction : Answer *all* questions.

PART – A

Answer **any five** questions.

(5×2 = 10)

1. a) Transform the matrix $\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$ into $\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$ using

elementary transformations.

b) Find the eigen values of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

c) Find the n^{th} derivative of $\log_e (3x - 2)$.

d) If $z = e^{\frac{x}{y}}$, find $\frac{\partial^2 z}{\partial x \partial y}$.

e) Evaluate $\int_0^{\pi} \sin^5 x \, dx$.

f) Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^2)^2} \, dx$.

g) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y - 2z - 3 = 0.$$

h) Find 'k' so that the spheres $x^2 + y^2 + z^2 + 4x + ky + 2z + 6 = 0$ and $x^2 + y^2 + z^2 + 2x - 4y - 2z + 6 = 0$ may be orthogonal.

P.T.O.



PART – B

Answer **one full** question.

(1×15 = 15)

2. a) Find rank of the matrix by reducing in to echelon form.

$$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$$

b) Solve completely the system of equations $x + 3y - 2z = 0$; $2x - y + 4z = 0$; $x - 11y + 14z = 0$.c) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

OR

3. a) Find the values of λ and μ such that the equations $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda = \mu$ have i) a unique solution ii) infinite number of solutions.b) Reduce the matrix $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{bmatrix}$ into normal form and find its rank.c) For the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, find A^{-1} using Cayley-Hamilton theorem.



PART - C

Answer two full questions.

(2×15 = 30)

4. a) Find the n^{th} derivative of $\frac{x^2}{(x-1)^2(x-2)}$.

b) Find the n^{th} derivatives of

i) $(x^2 + 1)e^{5x}$ ii) $\cos 3x \sin 4x$.

c) If $y = (\sin^{-1}x)^2$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.

OR

5. a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x + y + z}$.

b) State and prove Euler's theorem for homogeneous function of x and y .

c) Find $\frac{dz}{dt}$, if $z = \log(x^2 - y^2)$, where $x = e^t \cos t$, $y = e^t \sin t$.

6. a) If $u = f(y - z, z - x, x - y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

b) If $u = \frac{yz}{x}$, $v = \frac{xz}{y}$, $w = \frac{xy}{z}$ show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$.

c) For any positive integer n , prove that

$$\int_0^{\frac{\pi}{2}} \cos^n x dx = \frac{(n-1)(n-3)(n-5)\dots 2 \text{ or } 1}{n(n-2)(n-4)\dots 2 \text{ or } 1} \cdot R$$

Where $R = \frac{\pi}{2}$, if n is even

$= 1$, if n is odd.

OR



7. a) Obtain the reduction for $\int \sec^n x dx$

b) Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$.

c) Evaluate $\int_0^{\infty} \frac{e^{-x} \sin \alpha}{x} dx$, where α is a parameter using Leibnitz's rule of differentiation under integral sign.

PART – D

Answer **one full** question.

(1×15 = 15)

8. a) Find the equation of the plane passing through (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-5}{6} = \frac{z-6}{7}$ are coplanar and find the point of intersection.

c) Find the equation of sphere passing through the points (3, 0, 0), (0, -1, 0), (0, 0, -2) and having its centre on the plane $3x + 4z - 1 = 0$.

OR

9. a) Show that the lines $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y-2}{2} = \frac{z-6}{4}$ are coplanar. Find the equation of the plane containing these lines.

b) Derive the equation of a right circular cone in the standard form $x^2 + y^2 = z^2 \tan^2 \alpha$.

c) Find the equation of right circular cylinder whose radius is 4 units and passes through (1, -2, 3) and (3, -1, 1).
