SM – 376

Second Semester B.Sc. Examination, May/June 2018
(CBCS) (Fresh) (2017-18 and Onwards)
STATISTICS – II
Basic Statistics – 2

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **ten** sub-divisions from Section **A** and **five** questions from Section **B**.*

*2) Scientific calculators are **permitted.***

SECTION – A (**20** marks)

1. Answer **any ten** sub-divisions from the following : **(10×2=20)**

a) Define a random variable (r.v.) and distinguish between discrete r.v. and continuous r.v.

b) Define probability mass function (pmf) of a r.v. and state its properties.

c) If X is a r.v, a and b are constants, prove that $E(aX + b) = aE(X) + b$.

d) State the properties of moment generating function (m.g.f.) of a r.v.

e) Define binomial distribution. Mention the conditions for its convergence to Poisson distribution.

f) Comment on the following statements :

i) The mean and variance of Binomial distribution are 3 and 4 respectively.

ii) For Poisson distribution $E(X) = 8$ and $V(X) = 6$.

g) Find mean of exponential distribution with p.d.f. : $f(x) = \theta\, e^{-\theta x}$; $x > 0$ and $\theta > 0$.

h) If (X, Y) has joint p.d.f. : $f(x, y) = e^{-(x+y)}$; $x > 0$ and $y > 0$ find marginal p.d.f. of X.

i) If X and Y are two independent random variables, show that their covariance is zero.

j) Define conditional expectation.

k) Find "K" if $f(x, y) = K(x + y)$; $0 < x < 1$ and $0 < y < 2$ is the joint p.d.f. of (X, Y).

l) State central limit theorem for independent and identically distributed random variables.

P.T.O.

SECTION – B (**50** marks)

Answer **any five** questions from the following : **(5×10=50)**

2. a) Define distribution function of a random variable. State its properties and prove one of them.

   b) Prove the following :

      i) $E(aX) = aE(X)$

      ii) $V(aX) = a^2 V(X)$

      iii) $E(X - a) = E(X) - a$, where 'X' is a r.v. and 'a' is a constant.

   c) A r.v. X takes the values –2, –1, 0 and 1 with probabilities $\frac{1}{8}, \frac{1}{8}, \frac{1}{4}$ and $\frac{1}{2}$ respectively. Find the probability distribution of $Y = X^2 + 1$ and also cumulative distribution. **(4+3+3)**

3. a) Find mean and variance of Binomial distribution.

   b) Obtain the recurrence relation for central moments of Poisson distribution and hence obtain variance. **(5+5)**

4. a) Find mean of Hyper-Geometric distribution.

   b) Obtain m.g.f. of geometric distribution.

   c) State and prove additivity property of negative-binomial variates. **(3+3+4)**

5. a) Define continuous uniform distribution and obtain its mean and variance.

   b) Find the distribution function of exponential distribution with mean $\frac{1}{\theta}$.

   c) Define Cauchy distribution. **(5+3+2)**

6. a) Define Gamma distribution and establish its additivity property.

   b) Find mean and variance of Beta distribution of second kind. **(4+6)**

7. a) Obtain m.g.f. of normal distribution.

   b) For Normal distribution, show the all odd ordered central moments are zero and derive the expression for even ordered central moments. **(4+6)**

8. a) State and prove multiplication theorem of expectation for two random variables and generalize it.

   b) The joint p.d.f. of (X, Y) is $f(x, y) = k(x + y)$; $0 < x < 1$ and $0 < y < 2$. Find

      i) k

      ii) E(X)

      iii) V(X).

(4+6)

9. a) State and prove Chebyshev's inequality.

   b) Prove that Gamma distribution $G(\alpha)$ converges to normal distribution. **(5+5)**
