SM – 368

II Semester B.A./B.Sc. Examination, May/June 2018
(CBCS) (F+R) (2014-15 and Onwards)
MATHEMATICS (Paper – II)

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

1. Answer **any five** questions : **(5×2=10)**

a) On the set Q_1 the set of all rational numbers other than 1, $*$ is defined by $a * b = a + b - ab\ \forall\ a, b \in Q$. Find the identity element.

b) Prove that in a group G, $(a^{-1})^{-1} = a\ \forall\ a \in G$.

c) Find the angle between the radius vector and the tangent to the curve $r = ae^{\theta\cot\alpha}$.

d) Find the length of the polar subtangent for the curve $r = a\sec 2\theta$.

e) Find the asymptotes parallel to the coordinate axes for $(x^2 + a^2)\, y = bx^2$.

f) Find the length of an arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$.

g) Show that the equation $(x^2 - 2xy + 3y^2)\, dx + (y^2 + 6xy - x^2)\, dy = 0$ is exact.

h) Solve : $P^2 - 4P + 3 = 0$ where $P = \dfrac{dy}{dx}$.

PART – B

Answer **one full** question : **(1×15=15)**

2. a) Let G be the set of all rational numbers and $*$ be the binary operation on G defined by $a * b = \dfrac{ab}{7}\ \forall a, b \in G$ then prove that $(G, *)$ is an abelian group. Solve $4 * x = 3^{-1}$.

b) Prove that $G = \{1, 5, 7, 11\}$ is a group under multiplication modulo 12.

c) Prove that a non-empty subset H of a group $(G, *)$ is a subgroup of G, if and only if

i) $a * b \in H, \forall a, b \in H$

ii) $a^{-1} \in H, \forall a \in H$.

OR

P.T.O.

3. a) Prove that every group of order 4 is abelian.

   b) Prove that $G = \{2^n / n \in Z\}$ is a group under multiplication.

   c) Prove that $H = \{1, 2, 4\}$ is a subgroup of the group $G = \{1, 2, 3, 4, 5, 6\}$ under multiplication modulo 7.

PART – C

Answer **any two full** questions : **(2×15=30)**

4. a) With usual notations, prove that $\tan\phi = r\dfrac{d\theta}{dr}$ for the polar curve, $r = f(\theta)$.

   b) Show that the curves $r = a(1 + \cos\theta)$, $r = b(1 - \cos\theta)$ intersect orthogonally.

   c) Show that the evaluate of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$.

OR

5. a) Find the angle of intersection of the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$.

   b) Derive the formula for radius of curvature in parametric form.

   c) Find the pedal equation of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

6. a) Find all the asymptotes of the curve

   $2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$.

   b) Find the surface area of the solid generated by revolving about the y-axis the curve $x = y^3$ from $y = 0$ to $y = 2$.

   c) Find the position and nature of the double points of the curve

   $x^3 - y^2 + 4y - 7x^2 + 15x - 13 = 0$.

OR

7. a) Find the entire length of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$.

   b) Find the envelope of $x\cos^3\theta + y\sin^3\theta = c$, where θ is the parameter.

   c) Find the volume of the solid obtained by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line.

PART – D

Answer **any one full** question : **(1×15=15)**

8. a) Solve : $x\dfrac{dy}{dx} + (1 - x)\,y = x^2y^2$.

   b) Solve : $x = yP + P^2$.

   c) Verify for exactness and solve :

   $(4x + 3y + 1)\,dx + (3x + 2y + 1)\,dy = 0$.

OR

9. a) Solve : $x\dfrac{dy}{dx} + y\log y = xye^x$.

   b) Find the general and singular solution of $y = 3px + 6y^2p^2$. (Hint-put $y^3=v$).

   c) Show that the family of curves $\dfrac{x^2}{a^2+\lambda} + \dfrac{y^2}{b^2+\lambda} = 1$ is self orthogonal, λ is a parameter.