

III Semester B.Sc. Examination, Nov./Dec. 2017  
(CBCS) (F + R) (2015-16 and Onwards)

STATISTICS – III

Sampling Theory and Estimation



Time : 3 Hours

Max. Marks : 70

- Instructions :** 1) Answer **five** subdivisions from Section – A, **five** subdivisions from Section – B and **five** questions from Section – C.  
2) Answer Section – A in **first two** pages of answer book only.  
3) **Scientific** calculator is **allowed**.

SECTION – A

10

I. Answer **any five** sub-divisions from the following : (5×2=10)

- 1) a) Define population, finite population and infinite population.
- b) For SRSWOR, prove that  $E(\bar{y}) = \bar{Y}$ .
- c) In stratified random sampling, obtain an unbiased estimator of population mean  $\bar{y}$ .
- d) What is systematic sampling ? Explain.
- e) Define a sufficient estimator and give an example.
- f) Define efficient estimator and most efficient estimator.
- g) Define minimum variance bound estimator.
- h) Obtain moment estimator of  $\mu$  of Normal  $N(\mu, 1)$ .

SECTION – B

15

II. Answer **any five** sub-divisions from the following : (5×3=15)

- 2) a) What is sample survey ? Mention its limitations.
- b) Explain the method of selecting a random sample using random numbers.
- c) In SRSWOR, prove that

$$E[\hat{A}] = A \text{ where } \hat{A} = Np$$



- d) If  $X_1, X_2, \dots, X_n$  is a random sample from exponential distribution with mean  $\theta$ , then verify whether  $\bar{X}$  is an unbiased estimator of  $\theta$ .
- e) Obtain the efficiency of sample mean over sample median in estimating mean of normal population.
- f) Define the terms :
  - i) Standard error
  - ii) Mean square error.
- g) Obtain moment estimator of  $\theta$  of uniform distribution  $U(0, \theta)$ .
- h) Explain the general method of obtaining the confidence interval for the population parameter.

## SECTION – C

45

III. Answer **any five** questions from the following :

(5×9=45)

3) a) Explain sampling and non-sampling errors.

b) Obtain the sampling distribution of  $\frac{1}{\sigma^2} \sum_{i=1}^n (x_i - \bar{x})^2$ . (4+5)

4) a) In SRSWOR, prove that the probability of selecting  $i^{\text{th}}$  unit in the  $r^{\text{th}}$  draw is equal to the probability of its selection in the  $1^{\text{st}}$  draw.

b) Derive an expression for sample size under SRSWOR for estimating population mean  $\bar{Y}$ . (5+4)

5) What is allocation of sample size in stratified random sampling ? With usual notations obtain an expression for  $V(\bar{y}_{st})$  under proportional allocation and Neymann allocation. 9

6) Distinguish between linear and circular systematic sampling. Also prove

that  $V(\bar{y}_{sys}) = \frac{N-1}{N} S^2 - \frac{N-K}{N} S_{wsy}^2$ . 9

7) a) If  $X_1, X_2, \dots, X_n$  is a random sample from normal  $N(\mu, \sigma^2)$ ,  $\mu$  is known

then verify whether  $s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$  is consistent estimator for  $\sigma^2$ .



- b) For a random sample of size  $n$  from Poisson distribution  $P(\lambda)$ , verify whether  $\sum x_i$  is sufficient estimator of  $\lambda$ . (5+4)
- 8) a) State Crammer-Rao inequality. Obtain MVB estimator of  $P$  of Bernoulli distribution  $B(1, P)$ , based on a random sample of size  $n$ .
- b) Obtain an estimator of the parameter  $\sigma^2$  of normal  $N(\mu, \sigma^2)$  using maximum likelihood method ( $\mu$  is known). (4+5)
- 9) a) Obtain  $(1 - \alpha)$  100% confidence interval for the parameter  $P$  of Bernoulli population  $B(1, P)$  based on a random sample of size  $n$ .
- b) If  $X_{11}, X_{12}, \dots, X_{1n_1}$  is a random sample of size  $n_1$  from  $N(\mu_1, \sigma_1^2)$  and  $X_{21}, X_{22}, \dots, X_{2n_2}$  is another random sample of size  $n_2$  from  $N(\mu_2, \sigma_2^2)$ . Obtain  $(1 - \alpha)$  100% confidence interval for  $\sigma_1^2 / \sigma_2^2$ . (4+5)
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