



SN – 351



III Semester B.A./B.Sc. Examination, November/December 2017
(Semester Scheme) (CBCS) (F+R) (2015-16 and Onwards)
MATHEMATICS – III

Time : 3 Hours

Max. Marks : 70

Instruction: Answer *all* questions.

PART – A

1. Answer **any five** questions :

(5×2=10)

- Find the number of generators of the cyclic group of order 30.
- Define right coset and left coset of a group.
- Show that the sequence $\left\{\frac{1}{n}\right\}$ is monotonically decreasing sequence.
- State Raabe's Ratio test for convergence.
- Test the convergence of the series :

$$1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \dots$$

- Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in $[2, 4]$.
- State Cauchy's mean value theorem.
- Evaluate $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right)$.

PART – B

Answer **one full** question :

(1×15=15)

- If 'a and x' are any two elements of a group G then prove that $O(a) = O(x a x^{-1})$.
 - Let G be a cyclic group of order d and 'a' be a generator, then prove that the element a^k ($k < d$) is also a generator of G if and only if $(k, d) = 1$.
 - State and prove Fermat's theorem for groups.

OR

P.T.O.



3. a) Prove that if 'a' is any element of a group G of order n then $a^m = e$ for any integer m if and only if n divides m.
 b) Prove that every sub group of a cyclic group is cyclic.
 c) Prove that every group of order less than or equal to 5 is abelian.

PART – C

Answer **two full** questions :

(2×15=30)

4. a) Prove that the sequence $\left\{ \frac{2n-7}{3n+2} \right\}$
 i) is monotonically increasing
 ii) is bounded.
 b) Prove that a monotonic increasing sequence bounded above is convergent.
 c) Show that the sequence $\{x_n\}$ where $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ is convergent and converges to 2.

OR

5. a) Show that $\{a_n\} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}$ is convergent.
 b) Discuss the nature of the sequence $\{x^{1/n}\}$, $x > 0$.
 c) Examine the convergence of the sequences :

i) $\frac{(n+1)^{n+1}}{n^n}$

ii) $\left\{ \frac{2n^2 + 3n + 5}{n+3} \right\} \sin\left(\frac{\pi}{n}\right)$.

6. a) State and prove D'Alemberts Ratio test for series of positive terms.

b) Test the convergence of the series $1 + \frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots$



c) Sum the series to infinity $\frac{1}{5} - \frac{1.4}{5.10} + \frac{1.4.7}{5.10.15} - \frac{1.4.7.10}{5.10.15.20} + \dots$

OR

7. a) State and prove Cauchy's Root test for the convergence of series of positive terms.

b) Test the convergence of the series $\sum \frac{1.2.3.....n}{3.5.7.9.....(2n+1)}$.

c) Sum the series to infinity $\frac{1}{6} + \frac{1.4}{6.12} + \frac{1.4.7}{6.12.18} + \dots$

PART - D

Answer **one full** question :

(1×15=15)

8. a) Prove that a function, which is continuous in a closed interval, takes every value between its bounds at least once.

b) Evaluate $\lim_{x \rightarrow 0} \frac{e^{1/x}}{1 + e^{1/x}}$.

c) Evaluate $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$.

OR

9. a) Examine the differentiability of the function $f(x) = \begin{cases} x^2 - 1; & \text{for } x \geq 1 \\ 1 - x; & \text{for } x < 1 \end{cases}$

at $x = 1$.

b) State and prove Lagranges Mean value theorem.

c) Expand the function $\log_e(1+x)$ up to the term containing x^4 by Maclaurin's expansion.
