SN – 353

V Semester B.A./B.Sc. Examination, November/December 2017
(Semester Scheme) (CBCS) (2016 – 17 & Onwards)
(Fresh + Repeaters)
MATHEMATICS – V

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1 a) In a ring $(R, +, \cdot)$ prove that $\forall a, b, c \in R$, $a \cdot (b - c) = a \cdot b - a \cdot c$.

b) Show that the set of even integers is not an ideal of the ring of rational numbers.

c) Prove that every field is a principal ideal ring.

d) If $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, show that $\vec{F}$ is irrotational.

e) Find the maximum directional derivative of $x\sin z - y\cos z$ at (0, 0, 0).

f) Prove that $E\nabla = \nabla E = \Delta$.

g) Construct the Newton's divided difference table for the following data :

x	4	7	9	12
f(x)	– 43	83	327	1053

h) Using Trapezoidal rule to evaluate $\int_0^1 \frac{dx}{1+x}$ where

x	0	$1/6$	$2/6$	$3/6$	$4/6$	$5/6$	1
y = f(x)	1	0.8571	0.75	0.6667	0.6	0.5455	0.5

P.T.O.

PART – B

Answer **two full** questions : **(2×10=20)**

2. a) Prove that the set $R = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring with respect to '$\oplus_6$' and '$\otimes_6$' as the two compositions.

   b) Prove that a ring R is without zero divisors if and only if the cancellation laws hold in R

OR

3. a) Show that an ideal S of the ring of integers $(z, +, \cdot)$ is maximal if and only if S is generated by some prime integer.

   b) Prove that a commutative ring with unity is a field if it has no proper ideals.

4. a) If R is a ring and $a \in R$, let $I = \{x \in R / ax = 0\}$ prove that I is a right ideal of R.

   b) If $f : R \to R'$ be a homomorphism with kernel K, then prove that f is one-one if and only if $K = \{0\}$.

OR

5. a) Let $R = R' = C$ be the field of complex numbers. Let $f : R \to R'$ be defined by $f(z) = \bar{z}$ where $\bar{z}$ is the complex conjugate of z, show that f is an isomorphism.

   b) Prove that every homomorphic image of a ring R is isomorphic to some residue class (quotient) ring thereof.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Prove that $\nabla^2(f(r)) = f''(r) + \frac{2}{r} f'(r)$, where $r^2 = x^2 + y^2 + z^2$.

   b) Find the unit normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$.

OR

7. a) Show that Curl $\left[\vec{r}\times(\vec{a}\times\vec{r})\right]=3\vec{r}\times\vec{a}$ where $\vec{a}$ is constant vector and $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$.

b) If the vector $\vec{F}=(3x+3y+4z)\,\hat{i}+(x-ay+3z)\,\hat{j}+(3x+2y-z)\,\hat{k}$ is solenoidal, find ‘a’.

8. a) Prove that $\nabla^2\left(\frac{1}{r}\right)=0$, where $r^2=x^2+y^2+z^2$.

b) If $\vec{F}=\nabla(2x^3y^2z^4)$, find Curl $\vec{F}$ and hence verify that Curl $(\nabla\phi)=0$.

OR

9. a) If ϕ is a scalar point function and $\vec{F}$ is a vector point function, prove that

$$\text{div}(\phi\vec{F})=\phi\,\text{div}\,\vec{F}+\text{grad}\,\phi\cdot\vec{F}$$

b) Find Curl (Curl $\vec{F}$) if $\vec{F}=x^2y\,\hat{i}-2xz\,\hat{j}+2yz\,\hat{k}$.

PART – D

Answer **two full** questions : **(2×10=20)**

10. a) Use the method of separation of symbols to prove that

$$u_0+u_1x+u_2x^2+\dots\text{ to }\infty$$

$$=\frac{u_0}{1-x}+\frac{x\Delta u_0}{(1-x)^2}+\frac{x^2\Delta^2u_0}{(1-x)^3}+\dots\text{ to }\infty.$$

b) i) Evaluate $\Delta^{10}\left[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)\right]$.

ii) Express $f(x)=3x^3+x^2+x+1$ as a factorial polynomial (taking $h=1$).

OR

11. a) Find a second degree polynomial which takes the following data :

x	1	2	3	4
f(x)	–1	–1	1	5

b) Find f(1.9) from the following table :

x	1	1.4	1.8	2.2
f(x)	2.49	4.82	5.96	6.5

12. a) Using Lagrange's interpolation formula find f(6) for the following data :

x	2	5	7	10	12
f(x)	18	180	448	1210	2028

b) Using Simpson's $\frac{3}{8}^{th}$ rule evaluate $\int_0^{0.6} e^{-x^2} dx$ by taking 6 sub intervals.

OR

13. a) Following is the table of the normal weights of babies during the first few months of life.

Age in months	2	5	8	10	12
Weight in kgs	4.4	6.2	6.7	7.5	8.7

Estimate the weight of a baby of 7 months old using Newton's divided difference table.

b) Obtain an approximate value of $\int_0^6 \frac{dx}{1+x^2}$ by Simpson's $\frac{1}{3}^{rd}$ rule.

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MATHEMATICS – VI

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) Write the Euler's equation when f is independent of x.

b) Find the differential equation in which functional $\int_{x_1}^{x_2} (y^2 + x^2 y^1)\, ds$ assumes extreme values.

c) Define Geodesic on a surface.

d) Show that $\int_c (x+y)\,dx + (x-y)\,dy = 0$ where 'c' is simple closed path.

e) Evaluate $\int_0^a \int_0^b (x^2 + y^2)\,dx\,dy$.

f) Evaluate $\int_0^1 \int_0^2 \int_0^3 (x+y+z)\,dx\,dy\,dz$.

g) State Stoke's theorem.

h) Using Green's theorem show that the area bounded by simple closed curve C is given by $\int_C xdy - ydx$.

P.T.O.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

   b) Show that the equation of the curve joining the points (1, 0) and (2, 1) for $I = \int_1^2 \frac{1}{x}\sqrt{1+(y')^2}\,dx$ is a circle.

OR

3. a) Show that the general solution of the Euler's equation for the integral $I = \int_{x_1}^{x_2} \left(\frac{y'}{y}\right)^2 dx$ is $y = ae^{bx}$.

   b) Find the Geodesic on a surface of right circular cylinder.

4. a) If cable hangs freely under gravity from two fixed points, show that the shape of the curve is catenary.

   b) Find the extremal of the functional $I = \int_0^{\pi} \left((y')^2 - y^2\right) dx$ under the conditions $y = 0$, $x = 0$, $x = \pi$, $y = 1$ subject to the condition $\int_0^{\pi} y\,dx = 1$.

OR

5. a) Find the extremal of the integral $I = \int_0^1 (y')^2 dx$ subject to the constraint $\int_0^1 y\,dx = 1$ and having $y(0) = 0$, $y(1) = 1$.

   b) Find the extremal of the functional $\int_{x_1}^{x_2} \left(y^2 + (y')^2 + 2ye^x\right) dx$.

PART – C

Answer **two full** questions. (2×10=20)

6. a) Evaluate $\int_C \left(x^2 + 2y^2x\right)dx + \left(x^2y^2 - 1\right)dy$ around the boundary of the region defined by $y^2 = 4x$ and $x = 1$.

   b) Evaluate $\iint_R \left(x^2 + y^2\right)dydx$ over the region in the positive quadrant for which $x + y \le 1$.

OR

7. a) Evaluate $\int_0^a \int_0^{2\sqrt{ax}} x^2 dydx$ by changing the order of integration.

   b) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.

8. a) Evaluate $\int_{-a}^{a} \int_{-b}^{b} \int_{-c}^{c} \left(x^2 + y^2 + z^2\right)dz\,dy\,dx$.

   b) By changing into polar co-ordinates, evaluate $\iint_R \sqrt{x^2 + y^2}\,dxdy$, where R is a circle $x^2 + y^2 = a^2$.

OR

9. a) Find the volume bounded by the surface $z = a^2 - x^2$ and the planes $x = 0$, $y = 0$, $z = 0$, $y = b$.

   b) Evaluate $\iiint_R xyz\,dx\,dy\,dz$ by changing it to the cylindrical polar coordinates where R is region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $z = 1$ and the cylinder $x^2 + y^2 = 1$.

PART – D

Answer **two full** questions. **(2×10=20)**

10. a) Evaluate using Green's theorem in the plane for $\int_c (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$, where 'c' is boundary of the region enclosed by x = 0, y = 0 and x+y = 1.

b) Using Gauss-divergence theorem, show that :

i) $\iint_s \vec{r}.\hat{n}\,ds = 3v$ ii) $\iint_s \nabla r^2 \hat{n}\,ds = 6v$.

OR

11. a) State and prove Green's theorem.

b) Using Gauss-divergence theorem. Evaluate $\iint_s \vec{F}.\hat{n}\,ds$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and s is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.

12. a) Verify Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where s is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

b) Using Gauss divergence theorem evaluate $\iint_s (x\hat{i} + y\hat{j} + z^2\hat{k}).\hat{n}\,ds$ where s is closed surface bounded by cone $x^2 + y^2 = z^2$ and plane z = 1.

OR

13. a) Evaluate by Stoke's theorem $\int_c \sin z\,dx - \cos x\,dy + \sin y\,dz$, c is the boundary of the rectangle $0 \le x \le \pi, 0 \le y \le 1, z = 3$.

b) Verify Green's theorem for $\int_c (xy + y^2)\,dx + x^2 dy$ where c is the closed curve bounded by y = x and $y = x^2$.