SS – 345

V Semester B.A./B.Sc. Examination, Nov./Dec. 2018
(Semester Scheme)
(Fresh + Repeaters) (CBCS) (2016-17 and Onwards)
Mathematics
MATHEMATICS – V

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

1. Answer **any five** questions : (5×2=10)

a) In a ring $(R, +, \cdot)$, show that $a \cdot (-b) = (-a) \cdot b = -(a \cdot b)\ \forall a, b, \in R$.

b) Define subring of a ring and give an example.

c) Show that the set of even integers is an ideal of the ring of integers.

d) Find the unit normal vector to the surface $(x-1)^2 + y^2 + (z+2)^2 = 9$ at $(3, 1, -4)$.

e) If $\phi = 2x^3y^2z^4$, then find $\nabla\phi$.

f) Write the Newton's divided difference interpolation formula.

g) Evaluate $\Delta^{10}(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)$.

h) State the Trapezoidal rule for the integral $\int_a^b f(x)dx$.

PART – B

Answer **two full** questions. (2×10=20)

2. a) Prove that the intersection of any two subrings is a subring. Give an example to show that the union of two subrings of a ring need not be a subring.

b) Prove that $(z_5, +_5, \times_5)$ is a ring w.r.t. $+_5$ and $\times_5$.

OR

3. a) Prove that every field is an integral domain.

b) Show that the set of all real numbers of the form $a + b\sqrt{2}$, where a and b are integers is a ring w.r.to addition and multiplication.

P.T.O.

4. a) If $f : R \to R'$ be a homomorphism and onto then prove that f is one-one iff Ker $f = \{0\}$.

b) Prove that the set $S = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \Big/ a, b \in z \right\}$ of all 2×2 matrices is a left ideal of the ring R over Z. Also show that S is not a right ideal.

OR

5. a) State and prove fundamental theorem of homomorphism of rings.

b) Find all the principal ideals of the ring $R = \{0, 1, 2, 3, 4, 5, 6, 7\}$ w.r.to $+_8$ and $\times_8$.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Find the directional derivative of $\phi(x, y, z) = x^2 - y^2 + 4z^2$ at the point $(1, 1, -8)$ in the direction of $2\hat{i} + \hat{j} - \hat{k}$.

b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$.

OR

7. a) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$, where n is a non-zero constant. Also deduce that r^n is harmonic if $n = -1$.

b) If the vector $\vec{F} = (ax + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal, then find a.

8. a) If ϕ is a scalar point function and $\vec{F}$ is a vector point function. Then prove that $\text{div}(\phi\vec{F}) = \phi(\text{div}\vec{F}) + \nabla\phi \cdot \vec{F}$.

b) Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.

OR

9. a) Prove that :

i) Curl $\vec{F}$ is solenoidal.

ii) Grad ϕ is irrotational.

b) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r^2 = x^2 + y^2 + z^2$.

PART – D

Answer **two full** questions. (2×10=20)

10. a) By the separation of symbols, prove that

$$u_0 + \frac{u_1}{1!} + \frac{u_2 x^2}{2!} + \ldots\infty = e^x\left[u_0 + \frac{x\Delta u_0}{1!} + \frac{x^2\Delta^2 u_0}{2!} + \ldots\infty\right]$$

b) Obtain the function whose first difference is $6x^2 + 10x + 11$.

OR

11. a) From the following data find 'θ' at x = 84 using difference table.

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

b) Express $3x^3 - 4x^2 + 3x - 11$ in factorial notation. Also express its successive differences in factorial notation.

12. a) Prepare divided difference table for the following data.

x	1	3	4	6	10
f(x)	0	18	58	190	920

b) Evaluate $\int_0^6 \frac{1}{1+x^2}dx$, by using Simpson's $\frac{3}{8}^{th}$ rule.

OR

13. a) By using Lagrange interpolation formula find f(10) from the following data.

x	5	6	9	11
f(x)	12	13	14	16

b) Evaluate $\int_0^{0.6} e^{-x^2}dx$ by taking 6 sub intervals, by using Simpson's $\frac{1}{3}^{rd}$ rule.

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MATHEMATICS – VI

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

1. Answer **any five** questions. **(5×2 =10)**

a) Write Euler's equation when f is independent of y.

b) Find the differential equation of the functional $I = \int_{x_1}^{x_2} \left[y^2 - (y')^2 + 2ye^x\right] dx$.

c) Write the Euler's equation.

d) Evaluate $\int_C (3x + y)\,dx + (2y - x)\,dy$ along y = x from (0, 0) to (10,10).

e) Evaluate $\int_0^{\pi/2} \int_0^{a\cos\theta} r^2\,dr\,d\theta$.

f) Evaluate $\int_0^1 \int_0^2 \int_1^2 xyz\;dx\,dy\,dz$.

g) Find the area of the circle $x^2 + y^2 = a^2$ by double integration.

h) State Stoke's theorem.

PART – B

Answer **two full** questions. **(2×10 =20)**

2. a) Find the extremal of the functional $I = \int_0^{\pi/2} \left[y^2 - (y')^2 - 2y\sin x\right] dx$ under the end conditions $y(0) = y(\pi/2) = 0$.

b) Define Geodesic. Prove that geodesic on a plane is a straight line.

OR

3. a) If a cable hangs freely under gravity from two fixed points then show that the shape of cable is a catenary.

b) Solve the variational problem $\delta \int_0^{\pi/2} \left[y^2 - (y')^2 \right] dx = 0$ under the condition $y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 2$.

4. a) Prove that catenary is the curve which when rotated about a line generates a surface of minimum area.

b) Find the extremal of the functional $\int_{x_1}^{x_2} \left[12xy + (y')^2 \right] dx$.

OR

5. a) Find the extremal of the functional $\int_0^1 \left[x + y + (y')^2 \right] dx = 0$ under the conditions $y(0) = 1$ and $y(1) = 2$.

b) Find the extremal of the functional $\int_0^1 \left[(y')^2 + x^2 \right] dx$ subject to the constraint $\int_0^1 y\,dx = 2$ and having end conditions $y(0) = 0$ and $y(1) = 1$.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Evaluate $\int_C (x + y + z)\, ds$ where C is line joining the points (1, 2, 3) and (4, 5, 6) whose equations are $x = 3t + 1$, $y = 3t + 2$; $z = 3t + 3$.

b) Evaluate $\iint_R xy(x + y)\,dx\,dy$ over the region R bounded between the parabola $y = x^2$ and the line $y = x$.

OR

7. a) Change the order of integration in $\int_0^a \int_0^{2\sqrt{ax}} x^2\,dx\,dy$ and hence evaluate.

b) Evaluate $\iint_A \sqrt{4x^2 - y^2}\,dx\,dy$ where A is the area bounded by the lines $y = 0$, $y = x$ and $x = 1$.

8. a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dxdydz$.

b) Change into polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$.

OR

9. a) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration.

b) Evaluate $\iiint xyz\,dxdydz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by changing it to spherical polar coordinates.

PART – D

Answer **two full** questions. **(2×10 =20)**

10. a) State and prove Green's theorem.

b) Using divergence theorem, evaluate $\iint_S \left(x\hat{i} + y\hat{j} + z^2\hat{k}\right).\hat{n}ds$ where S is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane $z = 1$.

OR

11. a) By using divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface enclosing the region for which $x^2 + y^2 \le 4$ and $0 \le z \le 3$.

b) Evaluate $\iint \text{curl}\,\vec{F}.\hat{n}\,ds$ by Stoke's theorem if $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ and S is the surface of the cube $0 \le x \le 2, 0 \le y \le 2, 0 \le z \le 2$.

12. a) Using Green's theorem evaluate for the scalar line integral of $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ over the rectangular region bounded by the lines $x = 0, y = 0; x = a; y = b$.

b) Using the divergence theorem evaluate $\iint_S \vec{F}.\hat{n}ds$ where

$\vec{F} = (x^2 - yz)\,\hat{i} + (y^2 - zx)\,\hat{j} + (z^2 - xy)\,\hat{k}$ over the rectangular parallelopiped

$0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$.

OR

13. a) Using Green's theorem evaluate $\int_C (xy + y^2)\,dx + x^2dy$ where C is the closed curve bounded by $y = x$ and $y = x^2$.

b) Evaluate by Stoke's theorem $\oint_C yz\,dx + zx\,dy + xy\,dz$. where C is the curve $x^2 + y^2 = 1;\ z = y^2$.
