VI Semester B.A./B.Sc. Examination, May 2017
(Semester Scheme)
(Fresh) (CBCS) (2016-17 and Onwards)
MATHEMATICS – VII

Time : 3 Hours

Max. Marks : 70

Instruction : *Answer all Parts.*

PART – A

1. Answer **any five** questions : (5×2=10)

    a) Define a vector space over a field.

    b) For what value of K the vectors (1, 2, 3), (4, 5, 6) and (7, 8, K) are linearly dependent.

    c) Find the matrix of the linear transformation $T : R^2 \rightarrow R^3$ defined by
    $T(x, y) = (3x - y, 2x + 4y, 5x - 6y)$ w.r.t. the standard basis.

    d) Find the null space of the linear transformation $T : V_3(R) \rightarrow V_2(R)$ defined by
    $T(x, y, z) = (y - x, y - z)$.

    e) Write scalar factors in cylindrical co-ordinate system.

    f) Solve : $\dfrac{dx}{yz} = \dfrac{dy}{zx} = \dfrac{dz}{xy}$.

    g) Form a partial differential equation by eliminating the arbitrary constants from
    $z = (x + a)(y + b)$.

    h) Solve : $pq + p + q = 0$.

PART – B

Answer **two full** questions :   (2×10=20)

2. a) A subset W of a vector space V(F) is a subspace of V(F) if and only if

$C_1 \alpha + C_2 \beta \in W$, for $\alpha, \beta \in W$.

   b) Find the basis and dimension of the subspace spanned by

(1, – 1, 0), (0, 3, 1), (1, 2, 1) and (2, 4, 2) in V_3 (R).

OR

3. a) A set of non-zero vectors $(\alpha_1, \alpha_2, ...\alpha_n)$ of vector space V(F) is linearly

dependent if and only if one of vectors Say α_K (2≤ K ≤n) is expressed as a
linear combination of its preceding ones.

   b) Prove that W = {(x, y, z) | x = y = z} is a subspace of R^3.

4. a) If T : U→ V is a linear transformation then prove that

     i) T(0) = 0′ where 0 and 0′ are the zero vectors of U and V respectively.

     ii) T (– α) = – T(α), $\forall \alpha \in U$.

   b) Find the linear transformation T : $R^2 \to R^2$ such that T (1, 0) = (1, 1) and

T (0, 1) = (– 1, 2).

OR

5. a) State and prove rank nullity theorem.

   b) Show that the linear transformation T : $R^3 \to R^3$ given by T (e_1) = $e_1 + e_2$,

T (e_2) = $e_1 - e_2 + e_3$ and T(e_3) = 3e_1 + 4e_3 is non singular where {e_1, e_2, e_3}
is the standard basis of R^3.

PART – C

Answer **two full** questions :   (2×10=20)

6. a) Verify the condition of integrability and solve : 2yzdx + zxdy – xy (1 + z) dz = 0.

   b) Solve : (y – z) p + (z – x) q = x – y.

OR

7. a) Show that cylindrical coordinate system is orthogonal co-ordinate system.

b) Express the vector $\vec{f} = 3y\hat{i} + x^2\hat{j} - z^2\hat{k}$ in cylindrical co-ordinate and find f_ρ, f_ϕ, f_z.

8. a) Solve : $\dfrac{dx}{x(y-z)} = \dfrac{dy}{y(z-x)} = \dfrac{dz}{z(x-y)}$.

b) Solve : $\dfrac{dx}{y+z} = \dfrac{dy}{z+x} = \dfrac{dz}{x+y}$.

OR

9. a) Express the vector $\vec{f} = 3y\hat{i} + 2z\hat{j} + x\hat{k}$ in cylindrical co-ordinates and find f_ρ, f_ϕ, f_z.

b) Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in terms of spherical polar co-ordinates and find f_r, f_θ, f_ϕ.

PART – D

Answer **two full** questions :   (2×10=20)

10. a) Form partial differential equation by eliminating arbitrary function $f(xy + z^2, x + y + z) = 0$.

b) Solve : $p^2 - q^2 = x - y$.

OR

11. a) Solve $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = 12xy$.

   b) Solve : $z^2(p^2 + q^2) = x^2 + y^2$, by using the transformation $u = \dfrac{z^2}{2}$.

12. a) Find the complete integral of $z = pq$ by using Charpit's method.

   b) Solve : $\dfrac{\partial^2 z}{\partial x^2} - 7\dfrac{\partial^2 z}{\partial x \partial y} + 12\dfrac{\partial^2 z}{\partial y^2} = e^{x-y} + x^2$.

                         OR

13. a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \operatorname{Sin}^3\left(\dfrac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$.

   b) Solve : $\dfrac{\partial u}{\partial t} = 16\dfrac{\partial^2 u}{\partial x^2}$ subjected to the condition :

   i) $u(0, t) = 0$, $u(1, t) = 0$ for all t

   ii) $u(x;0) = x^2 - x$, $0 \le x \le 1$.

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VI Semester B.A./B.Sc. Examination, May 2017
(Fresh) (CBCS) (2016-17 and Onwards) (Semester Scheme)
MATHEMATICS – VIII

Time : 3 Hours

Max. Marks : 70

Instruction : *Answer all questions/Parts.*

PART – A

Answer **any five** questions. (5×2=10)

1. a) Evaluate $\lim\limits_{z \to e^{\frac{i\pi}{4}}} \left(\dfrac{z^2}{z^4 + z^2 + 1} \right)$.

   b) Show that $|z - (2 + 3i)| = 5$ represents a circle.

   c) Prove that $u = y^3 - 3x^2y$ is a harmonic function.

   d) Define cross ratio of four points.

   e) Show that $f(z) = \sin z$ is analytic.

   f) State Liouville's theorem.

   g) Find the real root of the equation $x^3 - x - 2 = 0$ over the interval (1.5, 2) upto two approximation by bisection method.

   h) Write Newton Raphson iterative formula.

PART – B

Answer **four full** questions : (4×10=40)

2. a) Show that the locus of $\arg\left(\dfrac{\overline{z}}{z}\right) = \dfrac{\pi}{2}$ is a line through the origin.

   b) Show that necessary condition for a function $f(z) = u(x, y) + i\, v(x, y)$ to be analytic.

   OR

3. a) Evaluate $\lim\limits_{z \to 2e^{\frac{i\pi}{6}}} \left(\dfrac{z^2 - 4}{z^3 + z + 5} \right)$.

   b) Show that $f(z) = \cos z$ is analytic and hence show that $f'(z) = -\sin z$.

P.T.O.

4. a) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x (\cos y - \sin y)$.

   b) Find the orthogonal trajectories of the families of curves $e^{-x} \cos y + xy = c$.

<div align="center">OR</div>

5. a) If $f(z) = u + iv$ is analytic function then show that $\left(\dfrac{\partial f(z)}{\partial x} \right)^2 + \left(\dfrac{\partial f(z)}{\partial y} \right)^2 = |f'(z)|^2$.

   b) Show that $u = e^x \sin y + x^2 - y^2$ is harmonic and find its harmonic conjugate.

6. a) Evaluate $\displaystyle\int_{(0, 1)}^{(2, 5)} (3x + y)dx + (2y - x)dy$ along the curve $y = x^2 + 1$.

   b) State and prove Cauchy's inequality theorem.

<div align="center">OR</div>

7. a) Evaluate $\displaystyle\int_C \dfrac{z + 4}{z^2 + 2z + 5} \, dz$ where C is $|z + 1 - i| = 2$.

   b) If $f(z)$ is analytic inside and on a simple closed curve C and a is a point within C then prove that $f^n(a) = \dfrac{n!}{2\pi i} \displaystyle\int_C \dfrac{f(z)}{(z - a)^{n+1}} \, dz$.

8. a) Show that $w = \dfrac{1}{z}$ transform a circle to circle or to a straight line.

   b) Discuss the transformation $w = \operatorname{Sin} z$.

<div align="center">OR</div>

9. a) Find the bilinear transformation which maps $z = \infty$, i, 0 onto $w = 0$, i, ∞ respectively.

   b) Show that the transformation $w = \dfrac{i - z}{1 + z}$ makes the x-axis of the Z-plane onto a circle $|w| = 1$ and the points in the half plane $y > 0$ on the points $|w| < 1$.

PART – C

Answer **two full** questions. (2×10=20)

10. a) Using bisection method find a real root of $x^3 - 3x^2 + 1 = 0$ correct to three places of decimal.

    b) Use Newton-Raphson method to find a real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places.

OR

11. a) Solve the equation $x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 7$ using Jacobi's iteration method to third approximation.

    b) Find the largest eigen value of the matrix $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by power method.

12. a) Using Taylor's series method find y at $x = 0.2$ correct to four decimal points given $\dfrac{dy}{dx} = x - y^2$ and $y(0) = 1$.

    b) Solve using Runge-Kutta method $\dfrac{dy}{dx} = x + y$ and $y(0) = 1$ for $x = 0(0.2)0.4$.

OR

13. a) Solve $\dfrac{dy}{dx} = x - y$ by Euler's modified method with $y(0) = 1$ for $x = 0.2$ correct to 4 places of decimals.

    b) Using Euler's method solve $\dfrac{dy}{dx} = x - y$ for $x = 0(0.1)0.5$ given $y = 1$ when $x = 0$. Verify with the exact solution.

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