SM – 370

VI Semester B.A./B.Sc. Examination, May/June 2018
(Fresh+Repeaters) (Semester Scheme)
(CBCS) (2016-17 and Onwards)
MATHEMATICS – VII

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

1. Answer **any five** questions. **(5×2=10)**

a) In a vector space V over F show that $c\cdot\alpha = 0 \Rightarrow c = 0$ or $\alpha = 0$.

b) Show that $W = \{(0, 0, z)/z \in R\}$ is a subspace of $V_3(R)$.

c) Show that the vectors $\alpha_1 = (1, 1, 0)$, $\alpha_2 = (1, 1, 0)$, $\alpha_3 = (1, 0, 0)$ are linearly independent.

d) Show that $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (x + y, x - y)$ is a linear transformation.

e) Write the relation between the Cartesian coordinates and cylindrical coordinates of a point.

f) Solve $\frac{dx}{zx} = \frac{dy}{yz} = \frac{dz}{xy}$.

g) Form a partial differential equation by eliminating arbitrary constants from $x^2 + y^2 = (z - c)^2 \tan^2\alpha$, where c and α are arbitrary constants.

h) Solve $\sqrt{p} + \sqrt{q} = 1$.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) Show that $V = \left\{ \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} \middle| \, x, y \in R \right\}$ is a vector space over R.

b) State and prove the necessary and sufficient condition for a nonempty subset W of a vector space V(F) to be a subspace of V.

OR

P.T.O.

3. a) If V is n-dimensional vector space, show that
   i) any n+1 vectors are linearly dependent.
   ii) no set of n – 1 vectors can span V.
   b) Find the basis and dimension of the subspace spanned by (1, –2, 3), (1, –3, 4), (–1, 1, –2) of the vector space $V_3(R)$.

4. a) Find the linear transformation $T : R^2 \rightarrow R^3$ such that T(1, 1) = (0,1, 2), T(–1, 1) = (2, 1, 0).
   b) Given the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$, find the linear transformation $T : V_2(R) \rightarrow V_3(R)$ relative to the bases $B_1 = \{(1, 1), (-1, 1)\}$, $B_2 = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$.

OR

5. a) Let $T : V_3(R) \rightarrow V_3(R)$ be a linear transformation such that T(1, 0, 0) = (1, 0, 2), T (0, 1, 0) = (1, 1, 0), T(0, 0, 1) = (1, –1, 0). Find the range, null space, rank nullity and hence verify rank-nullity theorem.
   b) Let $T : V \rightarrow W$ be a linear transformation. Then show that
   i) R(T) is a subspace of W
   ii) N(T) is a subspace of V
   iii) T is one-one if and only if N(T) = {0}.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Verify the condition for integrability and solve
   $z^2dx + (z^2 - 2yz)\,dy + (2y^2 - yz - xz)\,dz = 0$.
   b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.

OR

7. a) Show that spherical coordinate system is orthogonal curvilinear coordinate system.
   b) Express $\vec{f} = 3xi - 2yzj + x^2zk$ in cylindrical coordinates and find f_ρ, f_ϕ, f_z.

8. a) Solve $\frac{dx}{1+y} = \frac{dy}{1+x} = \frac{dz}{z}$.

b) Solve $\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)}$.

OR

9. a) Express $\vec{f} = 2xi - 2y^2j + xzk$ in cylindrical coordinates system and find f_ρ, f_ϕ, f_z.

b) Express $\vec{f} = xi + yj + zk$ in spherical coordinate system and find f_r, f_θ, f_ϕ.

PART – D

Answer **two full** questions. **(2x10=20)**

10. a) Form the partial differential equation by eliminating arbitrary functions from $lx + my + nz = \phi\,(x^2+y^2+z^2)$.

b) Solve $x(1 + y)p = y\,(1 + x)\,q$.

OR

11. a) Solve $(D^2 - 5DD' + 4D'^2)\,z = \sin\,(4x + y)$.

b) Solve $p^2 = z^2\,(1 - pq)$.

12. a) Solve by Charpits method $z^2(p^2 + q^2 + 1) = 1$.

b) Solve $[D^2 - DD' - 6(D')^2]z = xy$.

OR

13. a) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ given $u(0, t) = 0$, $u(l, t) = 0$, $u(x, 0) = k(lx - x^2)$, $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$.

b) Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$, given $u(0, t) = 0$, $u(1, t) = 0$, $\forall t$, $u(x, 0) = x^2 - x$, $0 \le x \le 1$.

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MATHEMATICS – VIII

Time : 3 Hours   Max. Marks : 70

Instruction : *Answer **all** the questions/Parts.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) Evaluate $\lim\limits_{z \to -i} \frac{z^2+1}{z^6+1}$.

   b) Prove that $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic.

   c) Define an analytic function and give an example.

   d) Define bilinear transformation.

   e) Show that f(z) = cosz is analytic.

   f) State Liouvilles' theorem.

   g) Find the real root of the equation $x^3 - 9x + 1 = 0$ in (2.9, 3) by bisection method.

   h) Using Newton-Raphson method, find the real root of $x^2 + 5x - 11 = 0$ in (1, 2) in one iteration only.

PART – B

Answer **four full** questions : **(4×10=40)**

2. a) Show that $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle.

   b) Prove that the necessary condition for a function f(z) = u(xy) + iv(xy) to be analytic is $u_x = v_y$ and $u_y = -v_x$.

OR

3. a) Evaluate $\lim_{z \to 1+i} \left[\frac{z^2 - z + 1 - i}{z^2 - 2z + 2}\right]$.

b) Show that $f(z) = ze^z$ is analytic.

4. a) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x(\cos y - \sin y)$.

b) Find the orthogonal trajectories of the family of curves $2e^{-x}\sin y + x^2 - y^2 = c$.

OR

5. a) If $f(z) = u + iv$ is analytic and ϕ is any differentiable function of x and y, show that $\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 = \left[\left(\frac{\partial \phi}{\partial u}\right)^2 + \left(\frac{\partial \phi}{\partial v}\right)^2\right] | f'(z) |^2$.

b) Show that $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.

6. a) Evaluate $\int_{(0,1)}^{(2,5)} (3x + y)\,dx + (2y - x)dy$ along

i) The curve $y = x^2 + 1$.

ii) The line joining (0, 1) and (2, 5).

b) State and prove fundamental theorem on algebra.

OR

7. a) Evaluate $\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)\ (z-2)} dz$ where C is a circle $|z| = 3$.

b) State and prove Cauchy's integral theorem.

8. a) Prove that the Bilinear transformation preserves the cross ratio.

b) Discuss the transformation $w = z^2$.

OR

9. a) Find the bilinear transformation which maps $z = 0, -i, -1$ on to $w = i, 1, 0$ respectively.

b) Show that the transformation $w = \frac{i - z}{i + z}$ makes the x-axis of the z-plane on to a circle $| w | = 1$ and the points in the half plane $y > 0$ on the points $| w | < 1$.

PART – C

Answer **two full** questions.

(2×10=20)

10. a) Find the root of the equation $x^3 - 4x + 1 = 0$ over (0, 1) by Regula-Falsi method.

b) Find the cube root of 24, correct to three decimal places by Newton-Raphson method.

OR

11. a) Solve the equation

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$ by Gauss-Seidel method.

b) Find the largest eigen value of the matrix and its corresponding eigen vector

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$ by power method.

12. a) Find the solution of $\frac{dy}{dx} = xy$ with $y(2) = 2$ at $x = 2.1$ correct to four decimal places, using Taylor series.

b) Solve $\frac{dy}{dx} = \frac{y - x}{y + x}$ with $y(0) = 1$ for $x = 0.1$ by Euler's method.

OR

13. a) Solve $\frac{dy}{dx} = x + y$ with $y(0) = 1$ for $x = 0.1$ using Euler's modified method.

b) Solve $\frac{dy}{dx} = xy$ given $y(1) = 2$ at $x = 1.2$ by Runge-Kutta method.